HYPERSONIC VISCOUS FLOW PAST A BLUNT CONE WITH MASSIVE INJECTION AT THE NOSE

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Introduction. One of the effective means of thermal protection of hypersonic flight vehicle is the use of massive injection. A number of studies (e.g., [1-4]) is devoted to the investigation of the associated gasdynamic problem. Asymptotic solutions to Navier–Stokes equations have been found in these studies in the neighborhood of the bluntness with the condition that $M_\infty >> 1$, $(\rho_\infty/\rho_1) << 1$, $Re_1 >> 1$, $(v_W/u_\infty) >> 1/\sqrt{Re_1}$, where M_∞ is the free stream Mach number, u_∞ is the free stream velocity, ρ_∞ is the free stream density, ρ_1 is the gas density behind the shock, Re_1 is the Reynolds number based on the free stream velocity and density, nose radius, and coefficient of viscosity at stagnation temperature, and v_W is the injection velocity at the body surface. Besides, a classification of possible flow conditions in the viscous hypersonic flow past blunt bodies is given in [4]. The flow is assumed to be laminar. The absorption of fluid injected in the neighborhood of the small bluntness by the boundary layer in the lateral surface of the cone is studied in the present paper. Laminar, viscous hypersonic flow past a blunt cone is considered. The fluid is injected in the neighborhood of the nose so that the boundary layer is displaced from the surface and becomes a mixing layer whose thickness is much less than that of the injected fluid. In its turn, the thickness of the injected fluid is much less than the thickness of the shock layer, and the flow in it is described by equations of inviscid boundary layer. The injected layer remains inviscid for a certain distance downstream and on the lateral surface of the cone where there is no injection. However, this layer of fluid is later absorbed by the boundary layer on the surface of the body and by the mixing layer at the contact boundary with the hot gas behind the shock wave.

1. Flow in the Neighborhood of the Blunt Nose. The flow in the neighborhood of the blunt nose is estimated (the region 0, Fig. 1). Let r be the nose radius; $\theta = O(1)$ is the semivertex angle of the cone; γ and ρ_1 are the adiabatic index and density behind the shock wave; μ_1 is the coefficient of viscosity at stagnation temperature; ρ_W, v_W, T_W, and γ_W are the density, velocity, temperature, and the adiabatic index of the injected fluid. The characteristic value of the pressure in the shock layer is $p_1 \sim \rho_\infty u^2_\infty$. The characteristic value of Reynolds number in the shock layer at the contact boundary is

$$\mathrm{Re}_1 = \rho_1 u_1 r/\mu_1 = \mathrm{Re}_0/\sqrt{\varepsilon},$$

where $u_1 \sim u_\infty\sqrt{\varepsilon}$, $\varepsilon = (\gamma - 1)/(\gamma + 1)$, $Re_0 = \rho_\infty u_\infty r/\mu_1$. Thus, the mixing layer thickness is

$$\delta_1 \sim r/\sqrt{\mathrm{Re}_1}.$$

It is possible to use Bernoulli's equation and the equation of state to obtain the following estimation for density and the streamwise velocity component u_W in the injected layer:

$$\rho_w/\rho_1 \sim \varepsilon/t\varepsilon_1,\ u_w \sim u_\infty\sqrt{t\varepsilon_1}, \tag{1.1}$$

where $\varepsilon_1 = (\gamma_W - 1)/(\gamma_W + 1)$; $t = 2C_{pW}T_W/u^2_\infty$ is the temperature factor.

The mass flow in the shock layer at the nose

$$\psi_\infty \sim \pi r^2 \rho_\infty u_\infty.$$

The mass flow of the injected fluid

$$\psi_w \sim \pi r^2 \rho_w v_w.$$

The nondimensional injection parameter is thus

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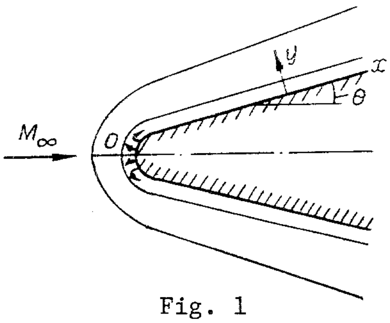

Fig. 1

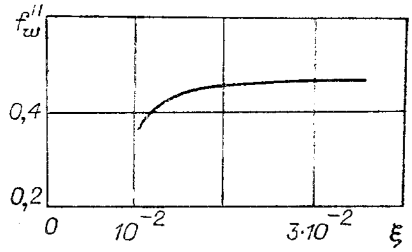

Fig. 2

$$g = \psi_w/\psi_\infty = (\rho_w v_w)/(\rho_\infty u_\infty).$$

If the continuity equation and relation (1.1) are used, then it is possible to make the following estimate for the thickness of the injected layer:

$$\delta_{\text{в}} \sim rg\sqrt{t\varepsilon_1}.$$

Stipulating the condition that the thickness of the injected layer is much less than the thickness of the shock layer $\delta \sim r\varepsilon$:

$$(\delta_{\text{в}}/\delta) \sim g\sqrt{t\varepsilon_1/\varepsilon^2} \ll 1.$$

On the other hand, if the thickness of the mixing layer δ_1 is required to be much less than the thickness of the injected layer:

$$(\delta_1/\delta_{\text{в}}) = 1/(g\sqrt{t\varepsilon_1 \mathrm{Re}_1}) \ll 1. \tag{1.2}$$

Here and in what follows we shall assume that $\varepsilon = O(1)$, $\varepsilon_1 = O(1)$, $t = O(1)$. In satisfying condition (1.2) the flow in the injected layer in the neighborhood of the nose will be inviscid, since the characteristic Reynolds number in the injected layer is

$$\mathrm{Re}_w = \mathrm{Re}_1\left(\frac{\mu_1}{\mu_w}\right)\Bigg/\sqrt{t\frac{\varepsilon}{\varepsilon_1}} \sim \mathrm{Re}_1 \tag{1.3}$$

and the ratio of the boundary layer thickness δ_W in the injected layer (where there is no injection) to the thickness of the mixing layer δ_1:

$$(\delta_w/\delta_1) \sim (t\varepsilon_1/\varepsilon)^{1/4}(\mu_w/\mu_1)^{1/2} = O(1). \tag{1.4}$$

Let the coordinate $\bar{x}$ be measured along the blunt surface of the cone, the coordinate $\bar{y}$ be measured normal to the cone surface. We introduce the following strained coordinates and asymptotic expansions for the flow in the injection layer at the nose:

$$\begin{gathered} \bar{x} = rx, \ \bar{y} = rg\sqrt{t\varepsilon_1}y, \\ \bar{u} = u_\infty\sqrt{t\varepsilon_1}\,u + \ldots, \ \bar{v} = u_\infty g t\varepsilon_1 v + \ldots, \\ \bar{p} = \rho_\infty u_\infty^2 p + \ldots, \quad \bar{\rho} = (\rho_\infty/t\varepsilon_1)\,\rho + \ldots \end{gathered} \tag{1.5}$$

If equations (1.5) are substituted in Navier–Stokes equations and the limiting approach is carried out,

$$g \to 0, \ \mathrm{Re}_1 \to \infty, \ \mathrm{M}_\infty \to \infty$$

under conditions (1.2)-(1.4), then it is possible to get the following system of equations for the inviscid boundary layer describing the flow in the injection layer:

$$\begin{gathered} \frac{\partial(\rho u b)}{\partial x} + \frac{\partial(\rho v b)}{\partial y} = 0, \\ \rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x}, \quad \frac{\partial p}{\partial y} = 0, \\ u\frac{\partial}{\partial x}\left(\frac{p}{\rho^{\gamma_w}}\right) + v\frac{\partial}{\partial y}\left(\frac{p}{\rho^{\gamma_w}}\right) = 0, \end{gathered} \tag{1.6}$$

where b is the distance from the axis of symmetry of the cone to its surface.

Boundary conditions for this system of equations have the form

$$v = v(x, 0),\ \rho = \rho(x, 0),\ u = 0 \tag{1.7}$$

with $0 \leq x \leq x_1$, $u = v = 0$ when $x > x_1$. The function $p(x)$ should be determined by matching the asymptotic expansions for the mixing zone and for the shock layer. This operation does not in any way differ from the matching of asymptotic expansions in [4].

Actually, if the above estimate for the flow in the neighborhood of the nose (region 0) is used, then it is possible to introduce the following strained coordinates and asymptotic expansions:

$$\begin{aligned} &\bar{x} = rx_0,\quad \bar{y} = r\varepsilon y_0 - \delta_B, \\ &\bar{u} = u_\infty u_0 + \ldots,\quad \bar{v} = \varepsilon u_\infty v_0 + \ldots, \\ &\bar{p} = \rho_\infty u_\infty^2 p_0 + \ldots,\quad \bar{\rho} = \varepsilon^{-1}\rho_\infty\rho_0 + \ldots \end{aligned} \tag{1.8}$$

After substituting these expressions in Navier—Stokes equations and carrying out the limiting solutions as $M_\infty \to \infty$, $Re_1 \to \infty$, and $\varepsilon = O(1)$, we get Euler's ordinary differential equations. The external boundary conditions for this system of equations are the Hugoniot conditions on the shock wave, and at $y_0 = 0$, $v = 0$. The solution to this system of equations ensures $u_0(x_0, y_0 \to 0) \sim \sqrt{\varepsilon}$ and $p_0(x_0, y_0 \to 0) \sim 1$.

The mixing layer whose thickness has been estimated above to be δ_1 is located at the boundary between the shock layer (region 0) and the injection layer. It is possible to introduce the following strained coordinates and asymptotic expansions for the mixing layer:

$$\begin{aligned} &x = rx_S, && \bar{y} = \delta_1 y_S - \delta_B, \\ &\bar{u} = u_\infty\sqrt{\varepsilon}u_S + \ldots, && \bar{v} = \delta_1 u_\infty\sqrt{\varepsilon}v_S + \ldots, \\ &\bar{p} = \rho_\infty u_\infty^2 p_S + \ldots, && \bar{\rho} = \varepsilon^{-1}\rho_\infty\rho_S + \ldots, \\ &\bar{H} = (u_\infty^2/2)H_S + \ldots, && \bar{\mu} = \mu_1\mu_S + \ldots, \end{aligned} \tag{1.9}$$

where δ_B is the thickness of the injected layer.

If these expressions are substituted in Navier—Stokes equations and the limiting solution $M_\infty \to \infty$, $Re_1 \to \infty$, $g \to 0$, $\varepsilon = O(1)$, $\varepsilon_1 = O(1)$, $t = O(1)$ is applied, then it is possible to obtain boundary layer equations for the flow in the mixing zone. Boundary conditions for these equations are found by matching asymptotic expansions (1.8) and (1.9), and we get $u_S(x_S, y_S \to \infty) = u_0(x_0, y_0 \to 0)$, $H_S(x_S, y_S \to \infty) = H_0(x_0, y_0 \to 0)$, $p_S(x_S) = p_0(x_0, y_0 \to 0)$. Matching asymptotic expansions (1.5) and (1.9), we find internal boundary conditions $u_S(x_S, y_S \to -\infty) = \sqrt{t\varepsilon_1}u(x, y \to 0)$, $H_S(x_S, y_S \to -\infty) = t$ and the magnitude of the pressure $p(x) = p_0(x_0) = p_S(x_S)$. The above analysis differs very marginally from [4] and hence it is in the nature of a summary.

If it is assumed that pressure $p(x)$ is specified, then it is possible to write the following solutions to the equations (1.6):

$$(p/\rho^{\gamma_w}) = C_1(\psi), \frac{u^2}{2} + \frac{\gamma_w}{\gamma_w - 1} p^{\frac{\gamma_w - 1}{\gamma_w}} C_1^{\frac{1}{\gamma_w}}(\psi) = C_2(\psi), \tag{1.10}$$

where ψ is the stream function. The functions $C_1(\psi)$ and $C_2(\psi)$ can be determined using boundary conditions (1.7). The particular form of the solution to (1.10) will be obtained below while considering the absorption of the injection layer by the boundary layer at the cone lateral surface. It is obvious that the injection layer remains inviscid even at a certain distance downstream from the injection region. Then, as a result of the effect of separation and growth of boundary layer thickness, the injection layer will be absorbed by the viscous flow at the lateral surface of the cone.

2. Absorption of Injection Layer by the Boundary Layer at the Lateral Surface of the Cone. Let the length along the generatrix of the cone on which the absorption of the injection layer by the boundary layer takes place, be $L = Kr$, where $K \gg 1$. The condition that

the fluid in the injection layer is completely absorbed by the boundary layer at the lateral surface of the cone, can be written in the form of the equation of mass flux:

$$\pi r^2 \rho_w v_w \sim 2\pi K r \sin\theta \rho_w u_w \delta_2,$$

where δ_2 is the boundary layer thickness at the side of the cone. All the estimates for flow variables obtained in the neighborhood of the nose remain valid even at the side surface of the cone because the semivertex angle of the cone $\theta = O(1)$. Hence it follows that $\delta_2 \sim \delta_w\sqrt{K}$. Thus we get the similarity parameter characterizing the absorption of the injection layer by the boundary layer:

$$\Delta_g = \frac{2\sin\theta K^{3/2}}{g\sqrt{\mathrm{Re}_1(\mu_1/\mu_w)\sqrt{t\varepsilon/\varepsilon_1}}}.$$

This parameter is the ratio of boundary-layer thickness to the thickness of the injection layer at the lateral surface of the cone when these quantities are of the same order: $\Delta_g = O(1)$. Hence it is possible to obtain an estimate of the characteristic length L over which absorption of the injection layer by the boundary layer and the mixing layer takes place:

$$L \sim r\left[\frac{g}{2\sin\theta}\sqrt{\mathrm{Re}_1\left(\frac{\mu_1}{\mu_w}\right)\sqrt{\frac{t\varepsilon}{\varepsilon_1}}}\right]^{2/3}. \tag{2.1}$$

At lengths an order of magnitude less than L in the equation (2.1), but more than r by an order of magnitude, the flow pattern remains the same as in the nose region. We note that the cone surface pressure along these distances and distances determined by Eq. (2.1) is the same as that for the sharp cone since the effect of bluntness affects the inviscid flow past a body of revolution only at certain amount of bluntness [5].

We choose l such that $r \ll l \ll L$, where L is determined from Eq. (2.1) and this means that $\Delta_g \to 0$. At these distances the inviscid outer flow behind the shock wave corresponds to hypersonic flow past a sharp cone. Estimates for flow parameters remain as before, but the characteristic length is l. Thus, the form of the asymptotic expansions is the same as in the region 0 [Eq. (1.8)], only the quantity r is replaced by l, Reynolds number is based on the length l. As a result, we obtain Euler equations describing the flow behind the shock wave where the excellent approximate formula for pressure is that of the hypersonic flow past a tangential wedge ($p = \sin^2\theta$).

Fluid, passing through the shock wave near the nose, forms an entropy layer at the side of the cone with a thickness of $(\delta_3/l) \sim \varepsilon K^{-2}$, $K = l/r$. The thickness of the fluid layer injected near the nose is $(\delta_{B1}/l) \sim g\sqrt{t\varepsilon_1}K^{-2}$ at the side of the cone. Thus, the entropy layer is much thicker than the injected layer and hence the entropy layer will not be absorbed by the mixing layer. The characteristic velocity in the entropy layer is $\sim u_\infty$ and the density is $\sim\rho_\infty/\varepsilon$.

Strained coordinates and asymptotic expansions in the entropy layer:

$$\begin{aligned} \bar{x} &= l x_3, & \bar{y} &= \delta_3 y_3 - \delta_{B1}, \\ \bar{u} &= u_\infty u_3 + \ldots, & \bar{v} &= (u_\infty \delta_3) v_3 + \ldots, \\ \bar{p} &= \rho_\infty u_\infty^2 p_3 + \ldots, & \bar{\rho} &= (\rho_\infty/\varepsilon)\rho_3 + \ldots \end{aligned} \tag{2.2}$$

If these expressions are substituted in Navier—Stokes equations, with $M_\infty \to \infty$, $\mathrm{Re}_1 \to \infty$, $\Delta_g \to 0$ it is possible to obtain the equations of inviscid boundary layer whose solution has the functional form (1.10). The initial conditions are obtained by matching asymptotic expansions (2.2) and (1.8), and matching pressure in the shock layer on a sharp cone we get $p_3 = \sin^2\theta$. The quantity δ_{B1} is found by matching expansions for the injection layer.

The thickness of the mixing layer and the wall boundary layer at distance l is on the order of $\delta_l \sim \delta_1\sqrt{K}$, $K = l/r$. Asymptotic expansions for the injection layer have the form (1.9) with the replacement of r by l and δ_1 by δ_l. The pressure in the injected layer is equal to the pressure in the entropy layer and the inner boundary conditions have to be found by matching asymptotic expansions for the injected and mixing layers.

The strained coordinates and asymptotic expansions for the injected layer at distances l have the form

$$\bar{x} = lx_{B1}, \qquad \bar{y} = \delta_{B1}y_{B1},$$
$$\bar{u} = u_\infty\sqrt{t\varepsilon_1}u_{B1} + \ldots, \quad \bar{v} = u_\infty\sqrt{t\varepsilon_1}\delta_{B1}v_{B1} + \ldots, \tag{2.3}$$
$$\bar{p} = \rho_\infty u_\infty^2 p_{B1} + \ldots, \ \bar{\rho} = (\rho_\infty/t\varepsilon_1)\rho_{B1} + \ldots$$

After substituting these expressions in Navier–Stokes equations with $M_\infty \to \infty$, $g \to 0$, $Re_1 \to -\infty$, $(l/r) \to \infty$, we get equations for the inviscid boundary layer whose solution is described by Eq. (1.10). Matching asymptotic expansions (2.2) and (2.3) we get the value of pressure in the injected layer. The initial conditions for these equations can be obtained by matching asymptotic expansions (1.5) for the injected layer at the nose with the asymptotic expansions (2.3).

All this analysis makes it possible to explain the problem of initial conditions for the equations describing the absorption of the injected layer at the cone lateral surface. Finally, we note that asymptotic expansions (2.2) for the entropy layer at the side of a blunt cone at distances l are valid even at distances L, determined by the expression (2.1), since at these distances the thickness of the entropy layer is much larger than the mixing layer.

In order to describe the absorption of the injected layer at the lateral surface of the cone by the boundary layer and the mixing layer, we introduce the following strained coordinates and asymptotic expansions:

$$\bar{x} = Lx_2, \qquad \bar{y} = \delta_2 y_2, \tag{2.4}$$
$$\bar{u} = u_\infty\sqrt{t\varepsilon_1}u_2 + \ldots, \qquad \bar{v} = u_\infty\sqrt{t\varepsilon_1}\delta_2 v_2 + \ldots,$$
$$\bar{p} = \rho_\infty u_\infty^2 p_2 + \ldots, \qquad \bar{\rho} = (\rho_\infty/t\varepsilon_1)\rho_2 + \ldots,$$
$$\bar{H} = (u_\infty^2/2)H_2 + \ldots, \qquad \bar{\mu} = \mu_1\mu_2 + \ldots$$

If relation (2.4) is substituted in Navier–Stokes equations and the limiting approach $K \to \infty$, $M_\infty \to \infty$, $Re_1 \to \infty$, $g \to 0$, $t = O(1)$, $\varepsilon = O(1)$, $\varepsilon_1 = O(1)$, $\Delta_g = O(1)$ is taken, then it is possible to obtain the following system of equations:

$$\frac{\partial(b_2\rho_2u_2)}{\partial x_2} + \frac{\partial(b_2\rho_2v_2)}{\partial y_2} = 0, \tag{2.5}$$
$$\rho_2\left(u_2\frac{\partial u_2}{\partial x_2} + v_2\frac{\partial u_2}{\partial y_2}\right) = -\frac{\partial p_2}{\partial x_2} + \frac{\partial}{\partial y_2}\left(\mu_2\frac{\partial u_2}{\partial y_2}\right), \ \frac{\partial p_2}{\partial y_2} = 0,$$
$$\rho_2\left(u_2\frac{\partial H_2}{\partial x_2} + v_2\frac{\partial H_2}{\partial y_2}\right) = \frac{\partial}{\partial y_2}\left[\frac{\mu_2}{\Pr}\frac{\partial H_2}{\partial y_2} + t\varepsilon_1\mu_2\left(1 - \frac{1}{\Pr}\right)\frac{\partial u_2^2}{\partial y_2}\right],$$
$$p_2 = p_2(h_2, \rho_2), \ \mu_2 = \mu_2(h_2),$$

b_2 is the distance from the cone axis to its surface.

Asymptotic expansions (2.4) and boundary-layer equations (2.5) are correct not only for the injected layer but also for the mixing zone, since its thickness is of the same order as that of the injected layer. Actually, the displacement thickness at the sides of the cone at a distance L is $\delta_f \sim L/\sqrt{Re_f}$, where $Re_f = Re_1K$, i.e, $Re_f \sim Re_wK$ and, on the ohter hand, since $t = O(1)$, $\varepsilon = O(1)$, $\varepsilon_1 = O(1)$, the velocity, density, and enthalpy are of the same order.

Boundary conditions at the cone surface have the form

$$y_2 = 0, \ u_2 = v_2 = 0, \ H_2 = t. \tag{2.6}$$

The outer boundary conditions for the system of equations (2.5) can be obtained by combining asymptotic series (2.4) with the inviscid solution for the flow past a sharp cone (2.1) (at lengths L):

$$y_2 \to \infty, \ u_2 \to \sqrt{\varepsilon/(t\varepsilon_1)}\cos\theta, \ p_2 = \sin^2\theta, \ H_2 \to 1. \tag{2.7}$$

The outer boundary conditions for the system of equations (2.5) can be obtained by matching the asymptotic expansions (2.4) with inviscid solution for the flow past the lateral surface

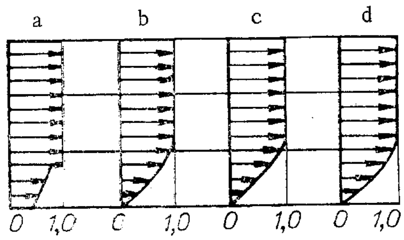

Fig. 3

of the cone by inviscid injected layer (2.3) and inviscid outer flow behind the shock wave (2.2), i.e., with the solution at distances l such that $r \ll l \ll L$. As a result we get

$$\left.\begin{aligned} u_2(0, y_2) &= u(y_2), \\ H_2(0, y_2) &= t, \end{aligned}\right\} \quad 0 \leqslant y_2 \leqslant y_{20}, \tag{2.8}$$
$$\left.\begin{aligned} u_2(0, y_2) &= \sqrt{\varepsilon/(t\varepsilon_1)}\cos\theta, \\ H_2(0, y_2) &= 1, \end{aligned}\right\} \quad y_2 > y_{20}.$$

Thus, the boundary-value problem (2.5)-(2.8) differs from the usual boundary-layer problem only by the presence of nonhomogeneous initial conditions (2.8).

As a computational example for the absorption of injected layer by the boundary layer the following case was chosen: cone semivertex angle $\theta = 45°$, $\gamma = 1.2$, $\gamma_w = 1.4$, temperature factor $t = 1$, and Prandtl number $Pr = 1$, change in viscosity with enthalpy is such that $\rho\mu$ = const, the constant is the same for both the fluids. In particular, the following type of injection at the spherical nose is chosen:

$$\rho(x,\ 0) = 1,\ v(x,\ 0) = 1,\ 0 \leqslant x \leqslant x_{10}, \tag{2.9}$$

where $\varphi_0 = \pi x_{10} = 45°$.

A good approximation for the pressure distribution at the nose surface is the Newton formula $p(x) = \cos^2 x$. Using this formula, and the boundary conditions (2.9), the solution to (1.10) is obtained in the form

$$u(x, \psi) = \sqrt{\frac{2\gamma_w}{\gamma_w - 1}\left[(1+\psi)^2 + (1+\psi)^{2/\gamma_w} p^{\frac{\gamma_w - 1}{\gamma_w}}(x)\right]},$$

where $\psi = \cos x' - 1$ is the stream function.

Let the formula for pressure at the cone lateral surface p_1 be given by $p_1 = \sin^2\theta$, which is valid for flow past a tangential wedge; then the velocity profile in the injected layer at the lateral surface of the cone has the form

$$u_1(x_1, \psi_1) = \sqrt{\frac{2\gamma_w}{\gamma_w - 1}\left[(1+\psi_1)^2 + (1+\psi_1)^{2/\gamma_w} (\sin\theta)^{\frac{2(\gamma_w - 1)}{\gamma_w}}\right]}.$$

If Dorodnitsyn—Lees transformation is used, the boundary-value problem (2.5)-(2.8) is reduced to the form

$$f''' + ff'' = 2\xi(f'f'^{\cdot} - f^{\cdot}f''), \tag{2.10}$$
$$f(0) = f'(0) = 0,\ f'(\infty) = 1,$$
$$f' = u_2/u_l,\ u_l = \sqrt{\frac{\varepsilon}{t\varepsilon_1}}\cos\theta,$$
$$\xi = \int\limits_0^{x_2} \rho_{2w}\mu_{2w}u_l b_2^2 dx_2,\ \eta = \frac{u_l b_2}{\sqrt{2\xi}}\int\limits_0^{y_2}\rho dy_2,\ (\,)' = \frac{\partial}{\partial\eta},\ (\,)^{\cdot} = \frac{\partial}{\partial\xi}.$$

In order to solve the boundary-value problem (2.10) with initial conditions (2.8), the procedure given in [6] was used. Computed results are given in Figs. 2 and 3. Figure 2 shows the skin friction distribution along the wall and Fig. 3 shows the growth of the velocity profile along the generatrix of the cone (a-d correspond to $\xi = 0$, 10^{-2}, $1.2 \cdot 10^{-2}$, and $3 \cdot 10^{-2}$). The absorption takes place very rapidly by the length scale L. In conclusion, it is worth noting that, in the general case, the displacement process will be accompanied by chemical reactions and, strictly speaking, the entire analysis given above is valid only when the Lewis number Le = 1.

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UNSTEADY MOTION OF A CIRCULAR CYLINDER IN A TWO-LAYER LIQUID

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We have performed a theoretical and experimental study of plane internal waves generated on the interface in a two-layer liquid by the unsteady translational motion of a submerged circular cylinder. At the present time wave formaton by such motion of a two-dimensional body has been analyzed theoretically only for the special case of a homogeneous liquid [1], and experimental research has been devoted mainly to the study of steady motion [2, 3].

We consider the linear formulation of the two-dimensional problem of wave flows generated by a dipole with a time-dependent moment moving in the upper layer of a two-layer liquid. We assume that, as in an infinite homogeneous liquid, this is equivalent to the motion of a circular cylinder of radius R with the velocity U(t) [the dipole moment $m(t) = 2\pi R^2 U(t)$, and coincides with the direction of motion of the cylinder]. We assume that the liquid is inviscid and incompressible, and consists of two layers of different densities: $\rho_1 (0 < y < H_1)$ and $\rho_2 = \rho_1(1 + \varepsilon)$, $\varepsilon > 0 (-H_2 < y < 0)$. The y axis is directed vertically upward, and the horizontal x axis lies in the undisturbed interface. We assume that at time t = 0 a dipole with the variable moment m(t) [$m(t) \equiv 0$ for $t \leq 0$] with its axis in the positive direction of the x axis begins to act in the upper layer of liquid at the point x = 0, y = h, so that its trajectory has the form x = c(t), y = h.

We assume potential flow in each layer, and that the equations of motion have the form

$$\Delta v_n = -\gamma_n m(t) \frac{\partial}{\partial x} \delta(x - c(t)) \frac{\partial}{\partial y} \delta(y - h)$$

with the boundary conditions

$$v_1 = 0 \quad \text{at} \quad y = H_1,$$

$$v_1 = v_2, \left[\rho_n \left(\frac{\partial^3 v_n}{\partial t^2 \partial y} - g \frac{\partial^2 v_n}{\partial x^2} \right) \right]_2^1 = 0 \quad \text{at} \quad y = 0,$$

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